TeV Scale Leptogenesis, θ_{13} And Doubly Charged Particles At LHC

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Abstract

We explore a realistic supersymmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model spontaneously broken at around 10^{12} GeV. The presence of D and F-flat directions gives rise to TeV mass doubly charged particles which can be found at the LHC. We implement TeV scale leptogenesis and employing both type I and II seesaw, the three light neutrinos are partially degenerate with masses in the $0.02 - 0.1$ eV range. The effective mass parameter for neutrinoless double beta decay is $0.03 - 0.05$ eV. We also find the interesting relation $\tan 2\theta_{13} \simeq \frac{\sin 2\theta_{12}}{\tan 2\theta_{23}} \left(\frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2} \right) \lesssim 0.02$.

It has been recognized for some time that spontaneously broken supersymmetric models with D and F-flat directions can lead to interesting phenomenological and cosmological consequences [1, 2, 3, 4, 5, 6, 7]. An intermediate symmetry breaking scale of order $10^8 - 10^{16}$ GeV is a characteristic feature of these models. Another important aspect is the appearance of thermal inflation typically involving about 10 or so e-foldings [2, 4, 6, 7]. The entropy generation associated with thermal inflation has been exploited to try to resolve the gravitino [2] and moduli problem [8], and to suppress the primordial monopole number density to acceptable levels [5]. The entropy production does have an important drawback though. It will dilute , and in some cases completely wash away, any pre- existing baryon asymmetry. This very depends on the magnitude of the intermediate scale M_I [6].

In [9], with an intermediate scale of order 10^8 GeV, the observed baryon asymmetry was explained via resonant leptogenesis [10]. The relatively low intermediate scale causes a moderate amount of dilution of an initially large lepton asymmetry, such that the final baryon asymmetry is consistent with the observations. This paper is partly motivated by the desire to implement TeV scale leptogenesis in models with an intermediate scale that is higher, namely of order 10^{12} GeV. (Scales significantly higher than this lead to a reheat temperature after thermal inflation that is too low for sphaleron transitions to be effective). To be specific, we base our discussion on the supersymmetric version of the well known gauge group $G_{221} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [11, 12]. The presence of D and F-flat directions means that the 'flaton' fields $\phi, \overline{\phi}$ with vevs $= M_I \sim 10^{12}$ GeV, have an associated mass scale M_s, the supersymmetry breaking scale, of the order of TeV. Being in the triplet representation of $SU(2)_R$, these fields contain doubly charged particles which turn out to have masses of order M_s. Hence, they should be found at the LHC.

Thermal inflation is driven by $\phi, \overline{\phi}$ and after it is over, the flatons produce TeV mass right-handed neutrinos associated with the first two families, whose subsequent decay leads via leptogenesis to the observed baryon asymmetry. (Because it has mass of order M_I, the third generation right-handed neutrino is not accessible at the TeV scale). Taking into account both type I [13] and type II [14] seesaw mechanism, the light neutrinos turn out to have partially degenerate [15] masses close to $0.02 - 0.1$ eV (depending upon the choice of M_s and M_I), consistent with solar and atmospheric neutrino mass scales and mixings. The third neutrino mixing angle is given by $\sin\theta_{13} \lesssim 0.01$, while the effective mass parameter associated with neutrinoless double beta decay is about $0.03 - 0.05$ eV.

The gauge symmetry G_{221} is broken at an intermediate scale M_I which arises from an interplay between the supersymmetry breaking scale $M_s \ll M_I$ and a cutoff scale M_*. Since G_{221} is broken to the gauge group $SU(2)_L \times U(1)_Y$ with the vevs of $\phi\,(1,3,-2)$ and $\overline{\phi}\,(1,3,2)$, a discrete Z_2 symmetry remains unbroken [16] which is precisely 'matter' parity. Consequently, the LSP is stable. To generate the scale M_I via D and F-flat directions, we employ a discrete symmetry $Z_4 \times Z_8$ which, among other things, prevents terms such as $\overline{\phi}\phi$ from appearing in the superpotential.

Consider the superpotential

$$
\begin{aligned}
W \supset \;& \kappa HH \frac{(\overline{\phi}\phi)^3}{M_*^5} + \frac{\lambda}{4}\frac{(\overline{\phi}\phi)^4}{M_*^5} + \gamma_{33} L_3^c L_3^c \phi + \gamma_{12} L_1^c L_2^c \phi \left(\frac{\overline{\phi}\phi}{M_*^2}\right)^2 \\
& + Y_{22} L_2 L_2^c H + Y_{33} L_3 L_3^c H + Y_{13} L_1 L_3^c H \frac{(\overline{\phi}\phi)}{M_*^2} + p L_1 L_1 \Delta_L + \frac{c}{M_*}\Delta_L \phi HH \\
& + \frac{a}{M_*}\overline{\Delta}_L \Delta_L \overline{\phi}\phi + \frac{b}{M_*}\overline{\phi}\,\overline{\Delta}_L HH \left(\frac{\overline{\phi}\phi}{M_*^2}\right)^2 + \frac{d}{M_*}\overline{\Delta}_L \overline{\Delta}_L \overline{\phi}\,\overline{\phi} \left(\frac{\overline{\phi}\phi}{M_*^2}\right)^2 ,
\end{aligned}
\tag{1}
$$

where $H \,(\equiv [H_u, H_d])$ is a bidoublet[1] higgs superfield $(2,2,0)$, $L_i\,(2,1,-1)$, $L_j^c\,(1,2,1)$ are the left-handed and right-handed lepton doublets ($i,j = 1,2,3$ are family indices) respectively, $\Delta_L\,(3,1,2)$ is a $SU(2)_L$ higgs triplet with conjugate superfield $\overline{\Delta}_L\,(3,1,-2)$ which will be needed later on to provide type II seesaw contribution to the light neutrino mass matrix. The term proportional to κ can help resolve the MSSM μ problem ($\mu \sim \kappa \left(\frac{M_I}{M_*}\right)^5 M_I$ (see also [9]), and is expected to be of order few hundred GeV). With κ and γ_{12} of comparable magnitudes, the dominant decay channel of ϕ is to first and second generation right-handed neutrinos, N_1, N_2, and this is useful in realizing TeV scale leptogenesis through $N_{1,2}$ decay.

Fields	ϕ	$\overline{\phi}$	L_1^c	L_2^c	L_3^c	H	L_1	L_2	L_3	Δ_L	$\overline{\Delta}_L$
Z_4	1	-1	1	1	1	i	i	$-i$	$-i$	-1	1
Z_8	ω^4	ω^6	ω	ω^7	ω^2	ω	ω^3	1	ω^5	ω^2	ω^4

TABLE I: Discrete charges of various superfields.

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[1] With only one bidoublet higgs, there will be no CKM mixings. However, G_{221} can be embedded in a bigger group such as $SO(10)$, and it is then possible to induce non-zero CKM mixings through some additional 'matter fields' [17]. Another possibility is to include loop contributions in association with supersymmetry breaking terms [18]. To simplify our presentation we will not address these possibilities here. Generation of the lepton-mixing matrix will be discussed later in the paper.

In Table I we list the discrete charges of the various superfields. (For the cosmology of spontaneously broken discrete symmetries the reader is referred to [7].)

The zero temperature effective scalar potential of ϕ (we use ϕ to also represent the scalar component of the superfield) along the D-flat direction with $\langle\phi\rangle = \langle\overline{\phi}\rangle^\dagger$, is given by

$$V(\phi) = \mu_0^4 - 2M_s^2\,|\phi|^2 + 2\frac{\lambda^2|\phi|^{14}}{M_*^{10}}\,, \tag{2}$$

where μ_0^4 is introduced to ensure that at the minimum $\langle\phi\rangle = M_I$, $V(M_I) = 0$, and $-2M_s^2\,|\phi|^2$ is the soft supersymmetry breaking term with $M_s \sim$ TeV. Here it is assumed that a positive supersymmetry breaking mass squared term generated at some superheavy scale can acquire a negative sign, via radiative corrections involving the superpotential coupling $\gamma_{33}L_3^c L_3^c \phi$, at a lower energy [19]. Minimization of the effective potential yields the intermediate scale,

$$M_I = |\langle\phi\rangle| = \left[\frac{M_s^2 M_*^{10}}{7\lambda^2}\right]^{1/12}\,. \tag{3}$$

We will see shortly that $M_I \simeq 10^{12}$ GeV and $M_* \simeq 5.5 \times 10^{13}$ GeV, for $M_s \simeq 5.5$ TeV are compatible with TeV scale lepton asymmetry, with partial conversion of the latter via sphalerons into the observed baryon asymmetry.

For non-zero temperature (T) the effective potential gets an additional contribution [20], $V_T(\phi)$ [9]. For $\phi \ll T$ the temperature-dependent mass term is $\sigma\,T^2|\phi|^2$, where $\sigma \simeq 0.14$. For $T > T_c = \sqrt{2/\sigma}M_s$ the potential

$$V(\phi) = \mu_0^4 + (-2M_s^2 + \sigma T^2)|\phi|^2 + 2\frac{\lambda^2}{M_*^{10}}\,|\phi|^{14}\,, \tag{4}$$

develops a minimum at $\phi = 0$, with $V(\phi = 0) = \mu_0^4 = \frac{12}{7}M_s^2 M_I^2$. For $\phi > T$, the temperature-dependent term is exponentially suppressed and $V(\phi)$ develops another minimum at $\phi = M_I$. $\phi = 0$ remains an absolute minimum for $\mu_0 \lesssim T \lesssim M_I$, but for $T \lesssim \mu_0$, the true minimum at M_I takes over. The dark energy density associated with the absolute minimum $(10^{-12}$ eV$^4)$ is irrelevant for our purpose [21].

Due to the false vacuum energy density μ_0^4 the universe experiences roughly $\ln(\mu_0/T_c) \sim 8$ e-foldings of thermal inflation. The flaton has mass m_ϕ of order $2\sqrt{6}M_s$ and it can decay into right-handed neutrinos (with mass M_N) via the superpotential coupling $\gamma_{12}L_1^c L_2^c \phi\left(\overline{\phi}\phi/M_*^2\right)^2$. The decay width is given by

$$\Gamma_\phi \simeq \frac{\gamma_{12}^2}{8\pi}m_\phi\left(\frac{M_I}{M_*}\right)^8 f_\phi = 0.04\,\gamma_{12}^2\,M_s\left(\frac{M_s}{\lambda M_I}\right)^{8/5} f_\phi\,, \tag{5}$$

where we have used Eq. (3) with m_ϕ in terms of M_s, and $f_\phi = (1 - 4M_N^2/m_\phi^2)^{3/2}$. The other decay width, $\Gamma_{\phi \to \tilde{H}\tilde{H}} \sim (\kappa^2/8\pi)(M_I/M_*)^{10}m_\phi$, is clearly suppressed compared to Γ_ϕ. Therefore the final temperature (T_f) after thermal inflation can be expressed as $T_f \simeq 0.3\sqrt{(\Gamma_\phi M_P)} \sim 0.06\,\gamma_{12}(M_s/\lambda M_I)^{4/5}\sqrt{(M_s f_\phi M_P)}$, where $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. To estimate T_f we need to know γ_{12} which can be estimated as follows.

From the superpotential in Eq. (1) the right-handed neutrino mass matrix M_R is given by

$$M_R = \begin{pmatrix} 0 & x & 0 \\ x & 0 & 0 \\ 0 & 0 & M \end{pmatrix}, \tag{6}$$

where $x = \gamma_{12}(M_I/M_*)^4 M_I = \gamma_{12}M_I^{1/5}M_s^{4/5}\left(1/\sqrt{7}\lambda\right)^{4/5}$ and $M = \gamma_{33}M_I$. Diagonalizing M_R, we have $M_R = U_{RR}^* M_R^{diag} U_{RR}^\dagger$, where

$$U_{RR} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot P, \tag{7}$$

with real and positive eigenvalues $|M_1| = |M_2| = |x|$ and $|M_3| = |M|$. The phases of M_i are absorbed in U_{RR} through the phase matrix $P = diag(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, e^{-i\alpha_3/2})$ with $\alpha_i = \arg(M_i)$. Obviously the decay of flaton to $N_{1,2}$ is possible if $m_\phi \gtrsim 2M_{1,2}$ i. e. $|\gamma_{12}| \lesssim \sqrt{6}(\sqrt{7}\lambda)^{4/5}\left(\frac{M_s}{M_I}\right)^{1/5}$.

Hence in this model we find the final temperature to be

$$T_f \simeq 90\,\text{GeV}\left(\frac{\gamma_{12}}{0.074}\right)\left(\frac{M_s}{5.5\,\text{TeV}}\right)^{13/10}\left(\frac{10^{12}\text{GeV}}{M_I}\right)^{4/5}\left(\frac{1}{\lambda}\right)^{4/5}, \tag{8}$$

with $M_N/m_\phi \simeq 0.3$. It is gratifying that T_f is in a range where the electroweak sphalerons [22] are able to convert some fraction of the lepton asymmetry into baryon asymmetry [23], which sets an upper bound on M_I of 10^{12} GeV for $M_s \sim$ few TeV. In Fig. (1), the variation of T_f with M_I (for fixed M_s and $M_{1,2} \simeq 0.3\,m_\phi$) is shown where Eq. (5) is used along with the constraint on $|\gamma_{12}|$. For example with $|\gamma_{12}| \sim 0.074$ (for $\kappa \sim |\gamma_{12}|$, the μ term is $\mathcal{O}(10^2)$ GeV), the masses of first two generations of right-handed neutrinos are of order 8 TeV corresponding to $T_f \simeq 90$ GeV.

We now consider the case where $N_{1,2}$ are produced by the direct non-thermal decay of the flaton field ϕ. The ratio of the number density of right-handed neutrino, n_N, to the entropy density s is given by

$$\frac{n_N}{s} \simeq \frac{3}{2}\frac{T_f}{m_\phi}B_r \,, \tag{9}$$

where B_r denotes the the branching ratio into the right-handed neutrino channel. The resulting total lepton asymmetry produced by the N_1, N_2 decay is $\frac{n_L}{s} = \sum_i \frac{n_N}{s}\epsilon_i$, where ϵ_i is the lepton asymmetry produced per ith right-handed neutrino decay.

Unlike thermal leptogenesis, there is no wash-out factor in this non-thermal scenario [24] corresponding to the lepton number violating 2-body scatterings mediated by right-handed neutrinos, as long as the light right-handed neutrino masses $|M_i| \gg T_f$ [25, 26]. The wash-out factor is proportional to e^{-z}, where $z = M_i/T_f$ [27] and for $z \gtrsim 10$ it can be safely neglected.

The CP asymmetry ϵ_i is given by [28]

$$\epsilon_i = \frac{1}{8\pi}\sum_{k\neq i} f\left(\frac{|M_k|^2}{|M_i|^2}\right) \frac{\mathrm{Im}\left[(h^\dagger h)^2_{ik}\right]}{(h^\dagger h)_{ii}} \,, \tag{10}$$

where

$$f(y) = \sqrt{y}\left[\frac{2}{1-y} - \ln\left(1 + \frac{1}{y}\right)\right] \,, \tag{11}$$

and $h = \frac{m'_D}{v}$ is the neutrino Yukawa coupling matrix in the basis where the right-handed neutrino mass matrix M_R is diagonal with real and positive eigenvalues and v is the electroweak scale vev ($\simeq 174\,\mathrm{GeV}$). This expression is valid in the limit where $||M_i| - |M_j|| > \Gamma_{N_i} + \Gamma_{N_j}$, where $\Gamma_{N_i} = \frac{1}{8\pi}(h^\dagger h)_{ii}|M_i|$ represents the decay width of the ith right-handed neutrino, and applies in our case.

The mass degeneracy between $|M_1|$ and $|M_2|$ can be broken by assuming the existence of new physics beyond $M_*(\simeq 5.5 \times 10^{13}$ GeV) which does not respect the discrete symmetry. Assuming this scale, M_G, to be near the GUT scale[2], an additional term in the superpotential such as $W_\xi = \eta L_2^c L_2^c \phi(\overline{\phi}\phi/M_G^2)$ can provide a suitable splitting ($\xi = \eta M_I(M_I/M_G)^2$), which can lead to the desired lepton asymmetry. To estimate the latter, let us assume that in the basis with M_R given by Eq. (6), the Dirac mass matrix m_D can be diagonalized by a

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[2] One may wonder about the origin of the two scales M_* and M_G. In a scenario with extra dimension(s) they can be associated with the compactification and cutoff scales respectively.

bi-unitary transformation

$$m_D = U_L^\dagger m_D^{diag} U_{RD},$$ (12)

which leads to

$$h^\dagger h = \frac{1}{v^2} U_R^\dagger \left(m_D^{diag}\right)^2 U_R,$$ (13)

where $U_R = U_{RD} U_{RR}$. We will consider $m_D^{diag} \equiv diag(m_{D_1}, m_{D_2}, m_{D_3}) = diag(m_e, m_\mu, m_\tau) \tan\beta$ which is possible within a left-right framework [29, 30]. The diagonal entries are taken to be real and positive. Notice that the left-handed rotation U_L is not present in Eq. (13). From Eq. (1), m_D is given by

$$m_D = \begin{pmatrix} 0 & 0 & \varepsilon_1 \\ 0 & Y_{22}v & 0 \\ 0 & 0 & Y_{33}v \end{pmatrix},$$ (14)

where $\varepsilon_1 = Y_{13}v \left(M_I/M_*\right)^2$. We find that $U_{RD} = I$, along with $m_{D_1} = 0$, $m_{D_2} \simeq Y_{22}v = m_\mu \tan\beta$ and $m_{D_3} \simeq Y_{33}v = m_\tau \tan\beta$ as previously mentioned. (Note that the electron mass vanishes in this approximation. It could arise via radiative corrections through flavor violating supersymmetry breaking contributions [31].) The deviation of U_L from identity matrix is parameterized by a small angle θ_{13_L} proportional to $\varepsilon_1/m_{D_3} \lesssim 10^{-2}$.

Substituting U_{RR} from Eq. (7) into Eq. (13), we find

$$\left(h^\dagger h\right)_{12} = \frac{-m_{D_2}^2}{2v^2} e^{i(\alpha_1 - \alpha_2)/2}; \quad \left(h^\dagger h\right)_{11} = \left(h^\dagger h\right)_{22} = \frac{m_{D_2}^2}{2v^2}.$$ (15)

In the limit of degenerate neutrinos $y \to 1$ and thus $f(y) \simeq 2/(1-y)$. From Eq. (10) we then have

$$\epsilon_1 \simeq \epsilon_2 \simeq \frac{1}{8\pi} \frac{|M_1|}{|M_1| - |M_2|} \frac{\text{Im}\left[\left(h^\dagger h\right)_{12}^2\right]}{(h^\dagger h)_{11}},$$

$$\simeq \frac{\Gamma_{N_1}}{|M_1|} \frac{|M_1|}{|M_1| - |M_2|} \sin(\alpha_1 - \alpha_2),$$ (16)

where we have used Eq. (15). Recall that the mass degeneracy between $|M_1|, |M_2|$ is removed by the introduction of the ξ term[3] with the superpotential W_ξ.

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[3] The introduction of new contributions like W_ξ and additional terms in m_D (to be considered later) which are important for light neutrino mixings, will not have much impact on Eqs. (15) and (16), and the results for lepton asymmetry will remain unaffected.

Note that the enhancement of the lepton asymmetry through the near degeneracy of $|M_1|$ and $|M_2|$ is restricted not only by the condition

$$||M_1| - |M_2|| \gtrsim 2\Gamma_{N_1} \,, \qquad (17)$$

but also by their nearly opposite CP parities. In the limit $|\xi| \ll |x|$, so that $\left|M_{1(2)}\right| \simeq |x|\left[1 - (+)\frac{|\xi|}{|x|}\cos(\delta_x - \delta_\xi) + \frac{1}{2}\frac{|\xi|^2}{|x|^2}\right]$, Eq. (16) can be translated[4] into [32]

$$\epsilon_1 \simeq \epsilon_2 \simeq \frac{\Gamma_{N_1}}{2\,|M_1|}\,\tan(\delta_x - \delta_\xi)\,, \qquad (18)$$

where $\delta_x - \delta_\xi = \frac{1}{2}\arg(\frac{x^2}{\xi^2})$. To achieve $|\xi| \ll |x|$, we need to consider $\eta \lesssim 10^{-2}$. Using equality condition [33] from Eq. (17), one can estimate the maximum value of $\epsilon = \epsilon_1 + \epsilon_2$ to be $\frac{|\xi|}{|M_1|}\sin(\delta_x - \delta_\xi) \simeq \frac{|\xi|}{|M_1|}$. The lepton asymmetry, with $B_r \sim 1$, is then

$$\frac{n_L}{s} \simeq \frac{3}{2}\frac{T_f}{m_\phi}\cdot 2\cdot\epsilon_1 \simeq \frac{3}{2}\frac{T_f}{m_\phi}\frac{\Gamma_{N_1}}{|M_1|}\,\tan(\delta_x - \delta_\xi)\,. \qquad (19)$$

From the observed baryon to photon ratio $n_B/n_\gamma \simeq (6.5 \pm 0.4) \times 10^{-10}$ [34] the lepton asymmetry is found to be $|n_L/s| \simeq (2.67 - 3.02) \times 10^{-10}$, where we have used $n_B/s \simeq (n_B/n_\gamma)/7.04$ [35] and $n_L/s = -(111/36)n_B/s$ [36] [5].

Before discussing the magnitudes of the parameters involved in Eq. (19) in order to be consistent with the observed n_B/s, let us first consider the light neutrino masses and related issues. From solar, atmospheric and terrestrial neutrino data (at 95% C.L.) [37], we have

$$\Delta m_\odot^2 \equiv \Delta m_{12}^2 = 7.92\,(1 \pm 0.09) \times 10^{-5}\ \mathrm{eV}^2\,, \qquad \sin^2\theta_{12} = 0.314\,(1_{-0.15}^{+0.18})\,;$$
$$\Delta m_{atm}^2 \equiv \Delta m_{23}^2 = 2.4\,(1_{-0.26}^{+0.21}) \times 10^{-3}\ \mathrm{eV}^2\,, \qquad \sin^2\theta_{23} = 0.44\,(1_{-0.22}^{+0.41})\,; \qquad (20)$$
$$\sin^2\theta_{13} = 0.9_{-0.9}^{+2.3} \times 10^{-2}\,.$$

Our next task is to make sure that the light neutrino mass matrix m_ν is consistent with Eq. (20). The lepton mixing matrix [38] is given by $U_{PMNS} = U_L U_\nu$, where U_L arises from the charged lepton sector, and U_ν comes from the diagonalization of m_ν, namely $m_\nu^{diag} = U_\nu^T m_\nu U_\nu$. Since $\theta_{13_L} \lesssim 10^{-2}$, the bilarge mixings must arise from U_ν. The structure of m_D given in Eq. (14) must be modified to generate appropriate atmospheric and solar

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[4] In this limit, $\sin(\alpha_1 - \alpha_2) \simeq \frac{|\xi|}{|x|}\frac{\sin(\delta_x-\delta_\xi)}{\left[1-\frac{|\xi|^2}{|x|^2}\cos^2(\delta_x-\delta_\xi)\right]}$.

[5] Here, the final temperature is just below the electroweak crossover scale, and we follow [36] to estimate the approximate conversion factor relating lepton and baryon asymmetries as $36/111 \simeq 0.324$. We thank the referee for raising this point.

neutrino mixings. Consider the terms $Y_{11}L_1L_1^cH(\overline{\phi}\phi/M_G^2)$ and $Y_{31}L_3L_1^cH(\overline{\phi}\phi/M_G^2)$ which contribute $\varepsilon_3 \equiv Y_{11}v\,(M_I/M_G)^2$ and $\varepsilon_2 \equiv Y_{31}v\,(M_I/M_G)^2$ to the 11 and 31 elements of m_D respectively. Being sufficiently small, they leave intact the relations $m_{D_2} \simeq Y_{22}v = m_\mu \tan\beta$, $m_{D_3} \simeq Y_{33}v = m_\tau \tan\beta$, $m_{D_1} \sim 0$ (to leading order). The deviation of U_{RD} from the identity matrix is parameterized by $\theta_{13_R} \sim \varepsilon_2/m_{D_3} \ll 1$.

Including the type II seesaw contribution to the neutrino mass matrix from the induced vev of Δ_L, we have[6]

$$m_\nu \simeq a_\Delta - m_D M_R^{-1} m_D^T,$$

$$\simeq \begin{pmatrix} a_\Delta + \frac{\xi}{x^2}\varepsilon_3^2 & -\frac{m_{D_2}}{x}\varepsilon_3 & \frac{\xi}{x^2}\varepsilon_2\varepsilon_3 - \frac{m_{D_3}}{M_I}\varepsilon_1 \\ -\frac{m_{D_2}}{x}\varepsilon_3 & 0 & -\frac{m_{D_2}}{x}\varepsilon_2 \\ \frac{\xi}{x^2}\varepsilon_2\varepsilon_3 - \frac{m_{D_3}}{M_I}\varepsilon_1 & -\frac{m_{D_2}}{x}\varepsilon_2 & \frac{\xi}{x^2}\varepsilon_2^2 - \frac{m_{D_3}^2}{M_I} \end{pmatrix}, \qquad (21)$$

where, for simplicity, $a_\Delta = 2\,p\,\langle\Delta_L\rangle \simeq 2\,p\,(cd/|a|^2)(v^2/M_I)(M_I/M_*)^4$ is taken to be real. Note that the terms proportional to ξ (with $\eta \sim 10^{-2}$, $|\xi| \sim 100$ GeV) are accompanied by factors $\varepsilon_i\varepsilon_j/x^2$, $i,j = 2,3$, and can be safely ignored. As for the lepton asymmetry, only the relative phase between ξ and x will be important.

To obtain the mass eigenstates, we first rotate m_ν by $U' = U_{23}U_{13}$ (where U_{ij} denotes the rotation matrix in the ij sector and we will ignore CP violation here) and express the effective mass matrix $\widetilde{m}_\nu = U'^T m_\nu U'$ in the new basis as

$$\widetilde{m}_\nu \simeq \begin{pmatrix} a_\Delta c_{13}^2 + 2\rho_1 s_{13}c_{13} + \Lambda_+ s_{13}^2 & \rho_2 c_{13} & 0 \\ \rho_2 c_{13} & \Lambda_- & \rho_2 s_{13} \\ 0 & \rho_2 s_{13} & a_\Delta s_{13}^2 - 2\rho_1 s_{13}c_{13} + \Lambda_+ c_{13}^2 \end{pmatrix}, \quad (22)$$

where

$$\Lambda_{+(-)} = -\frac{m_{D_3}^2}{M_I}c_{23}^2(s_{23}^2) - (+)2\frac{m_{D_2}}{x}\varepsilon_2 c_{23}s_{23},$$

$$\rho_{1(2)} = \frac{m_{D_3}}{M_I}\varepsilon_1 c_{23}(s_{23}) + (-)\frac{m_{D_2}}{x}\varepsilon_3 s_{23}(c_{23}), \qquad (23)$$

and $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Furthermore,

$$\tan 2\theta_{23} = \frac{2m_{D_2}\varepsilon_2}{x}\frac{M_I}{m_{D_3}^2}, \qquad (24)$$

$$\tan 2\theta_{13} = \frac{2\rho_1}{a_\Delta - \Lambda_+}. \qquad (25)$$

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[6] The scalar triplets in Δ_L are heavy with mass $\sim aM_I(M_I/M_*)$. Their decay does not contribute to the lepton asymmetry which requires two pairs of such triplets [39].

Note that for $2m_{D_2}\varepsilon_2/x \gg m_{D_3}^2/M_I$, 23 mixing can be maximized. An approximate diagonalization of m_ν is achieved by focusing on the 12 block of $\widetilde{m}_\nu$ and noting that θ_{13} is relatively small (see Eq. (20)). With $\rho_1 \ll (a_\Delta - \Lambda_+)$, to a good approximation the third state in Eq. (22) decouples. The upper left 2×2 block of $\widetilde{m}_\nu$ is readily diagonalized and the resulting mass eigenvalues are

$$m_{\nu_{1,2}} \simeq \frac{1}{2}\left[(a_\Delta + \Lambda_-) \mp \sqrt{(\Lambda_- - a_\Delta)^2 + 4\rho_2^2}\right] ; \quad m_{\nu_3} \simeq \Lambda_+ , \tag{26}$$

with

$$\tan 2\theta_{12} \simeq \frac{2\rho_2}{\Lambda_- - a_\Delta} . \tag{27}$$

Barring cancellation between a_Δ and Λ_- a large but non-maximal mixing angle θ_{12} is possible. The light neutrinos turn out to be partially degenerate ($|m_{\nu_1}| \sim |m_{\nu_2}| \sim |m_{\nu_3}| \gtrsim \sqrt{\Delta m_{\text{atm}}^2}$).

- The above consideration does not alter the requirement of $\tan\theta_{13}$ being small as $a_\Delta \sim \Lambda_- = -\Lambda_+ \tan^2\theta_{23}$. Here we have used compact forms of Λ_+ and Λ_- with the help of Eq. (24): $\Lambda_+ = -(m_{D_2}/x)\varepsilon_2 \cot\theta_{23}$; $\Lambda_- = (m_{D_2}/x)\varepsilon_2 \tan\theta_{23}$. Hence Eq. (25) can be approximated as

$$\tan 2\theta_{13} \simeq \rho_1 \frac{x}{m_{D_2}\varepsilon_2} \sin 2\theta_{23}. \tag{28}$$

ε_1 (GeV)	ε_2 (GeV)	ε_3 (GeV)	a_Δ (GeV)	x (TeV)	$\tan\beta$	$\tan 2\theta_{12}$	$\tan 2\theta_{23}$	$\tan 2\theta_{13}$
5.75×10^{-3}	1.67×10^{-6}	1.74×10^{-8}	4.38×10^{-11}	8.3	3.15	2.36	5.4	7×10^{-3}

TABLE II: A viable set of values for ε_i and the corresponding mixing angles (using $m_\mu \simeq 0.083$ GeV and $m_\tau \simeq 1.4$ GeV at M_I [40])

Table II presents a set of parameters and the corresponding mixing angles for $M_I = 10^{12}$ GeV and $M_s = 5.5$ TeV. (To achieve $a_\Delta \sim 0.044$ eV, we take $|a| \sim 1.25 \times 10^{-2}$ and $c \sim d \sim p \sim \mathcal{O}(1)$). The mass splittings are given by

$$\Delta m_\odot^2 \equiv \Delta m_{12}^2 = \left||m_{\nu_1}|^2 - |m_{\nu_2}|^2\right| \simeq (a_\Delta^2 - \Lambda_-^2)\sqrt{1 + \tan^2 2\theta_{12}} ,$$
$$\simeq 4a_\Delta \frac{|\rho_2|}{\sin 2\theta_{12}} , \tag{29}$$

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{23}^2 = \left||m_{\nu_3}|^2 - |m_{\nu_1}|^2\right| \simeq \left|\Lambda_+^2 - \Lambda_-^2 - \rho_2^2 + \frac{1}{2}\Delta m_\odot^2\right|,$$

$$\simeq 2a_\Delta \frac{m_{D_2}\varepsilon_2}{x}\frac{\cot 2\theta_{23}}{\sin^2\theta_{23}} + \mathcal{O}(\Delta m_\odot^2), \tag{30}$$

where we have used Eqs. (23), (24), (26) and (27).

As the dominant contributions to $\rho_{1,2}$ come from the second term in their expressions, we have $|\rho_2| \simeq |\rho_1|\cot\theta_{23}$. Using Eq. (28) we find

$$\tan 2\theta_{13} \simeq \frac{\sin 2\theta_{12}}{\tan 2\theta_{23}}\left(\frac{\Delta m_\odot^2}{\Delta m_{\text{atm}}^2}\right) \lesssim 0.02. \tag{31}$$

- The 13 mixing angle is well below the upper limit allowed by experiments. This is due to the fact that $\Delta m_\odot^2$ depends upon $|\rho_2|$ and in turn on $\tan 2\theta_{13}$. Higher values of $\tan\theta_{13}$ near the experimental upper limit cannot reproduce the appropriate $\Delta m_\odot^2$. With the parameters in Table II, the mass-squared differences are $\Delta m_\odot^2 \sim 7.6 \times 10^{-5}$ eV^2 and $\Delta m_{\text{atm}}^2 \sim 2 \times 10^{-3}$ eV^2. In Fig. (2) the allowed region for $\varepsilon_2, \varepsilon_3$ is shown for fixed M_s and M_I.

- For simplicity we have taken Y_{13} associated with ε_1 to be $\lesssim 10^{-1}$ and the relation (31) holds to a good accuracy. We have checked numerically that $Y_{13} \sim \mathcal{O}(1)$ would not change anything except that somewhat higher values of ε_3 ($\sim 3.3 \times 10^{-8}$ GeV) are allowed.

- Finally, θ_{13_L} induces a tiny correction to θ_{12} and θ_{23}. θ_{13} receives a correction of order $\theta_{13_L}c_{23}$ which could be significant for Y_{13} of order unity, i.e. $\sin\theta_{13}^{\text{eff}} \simeq \sin\theta_{13} - (\varepsilon_1/m_{D_3})c_{23}$. Even for this case the prediction for θ_{13} remains unaltered.

From Eq. (26) we find that the light neutrino masses are of the same order, close to $0.02-0.1$ eV. Figs. (3) and (4) display the range of allowed values for m_{ν_1} and m_{ν_3}. Following [15] these are partially degenerate neutrinos. This range of neutrino masses is below the so-called quasi-degenerate case. Furthermore, the effective mass parameter in neutrinoless double beta decay (which is the ee element ($\equiv a_\Delta$) of the neutrino mass matrix) [41] is estimated to be of order $0.03 - 0.05$ eV, corresponding to $M_I = 10^{12}$ GeV and $M_s = 5.5$ TeV.

We have checked that renormalization effects [42] do not alter our conclusions in any significant way. The estimated splitting between N_1 and N_2 due to running from M_* to

$M_{1,2}$ is of order $\frac{m_{D_2}^2}{4\pi^2 v^2}\ln\left(\frac{M_*}{10^4\text{GeV}}\right)M_2$, which is much smaller than the contribution arising from the term proportional to ξ. Furthermore, the running in m_D can be absorbed through a rescaling of m_{D_i}.

With the specified range of parameters involved, we are now in a position to calculate n_L/s from Eq. (19). Table III presents a sample value of the phase involved in n_L/s which is required to produce correct amount of lepton asymmetry. All other parameters are taken from Table II.

γ_{12}	$\tan(\delta_x - \delta_\xi)$	$\frac{n_L}{s}$
0.074	1.24	2.8×10^{-10}

TABLE III: Parameter values used in order to produce the required lepton asymmetry.

An important feature of our model is the existence of TeV scale doubly charged particles [43]. Writing

$$\phi = \begin{bmatrix} \frac{\phi^-}{\sqrt{2}} & \phi^0 \\ \phi^{--} & -\frac{\phi^-}{\sqrt{2}} \end{bmatrix} \text{ and } \overline{\phi} = \begin{bmatrix} \frac{\overline{\phi}^+}{\sqrt{2}} & \overline{\phi}^{++} \\ \overline{\phi}^0 & -\frac{\overline{\phi}^+}{\sqrt{2}} \end{bmatrix}, \tag{32}$$

and letting $\phi^0 = M_I + \eta/\sqrt{2}$ and $\overline{\phi}^0 = M_I + \overline{\eta}/\sqrt{2}$ ($\eta = \overline{\eta}^*$ is the real flaton field), the mass-squared matrix for the doubly charged particle is

$$
\begin{array}{cc}
 & \begin{array}{cc} \phi^{++} & \overline{\phi}^{++} \end{array} \\
\begin{array}{c} \phi^{--} \\ \overline{\phi}^{--} \end{array} & \begin{pmatrix} \frac{1}{7}M_s^2 + M_s^2 & \frac{6}{7}M_s^2 \\ \frac{6}{7}M_s^2 & \frac{1}{7}M_s^2 + M_s^2 \end{pmatrix},
\end{array}
\tag{33}
$$

where Eq. (3) has been used[7] and the soft masses are $M_s^2\,[\text{Tr }(\phi^\dagger\phi + \overline{\phi}^\dagger\overline{\phi})]$. Hence, the lightest doubly charged particles have masses $\sim \sqrt{2/7}M_s \sim 2.9$ TeV and can be found at the LHC [44]. The heavier doubly charged particles have masses of order 8 TeV. Note that the signs of the soft masses are important to keep the eigenvalues of this mass squared matrix positive. The existence of these light doubly charged states can also be inferred from the presence of an associated higher symmetry of the superpotential in Eq. (1) which leads to pseudo-Goldstone bosons. Finally, we note that in addition to the full MSSM spectrum

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[7] The contributions from the D terms turn out to be quite small.

of fields, the model contains a new singly charged field of mass $\sqrt{2}M_s \simeq 8$ TeV. There is yet another singly charged field with mass of order M_I, well beyond the reach of LHC.

In summary, we have presented a realistic supersymmetric model with gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, broken at an intermediate scale $M_I \sim 10^{12}$ GeV. Thermal inflation is followed by TeV scale leptogenesis. To reproduce the observed baryon asymmetry, the two lightest right-handed neutrinos are closely degenerate in mass, with $M_1 \simeq M_2 \sim 10^4$ GeV, while the mass of the third right-handed neutrino is $M_3 \sim 10^{12}$ GeV. The physics of neutrino oscillations requires both type I and type II seesaw, and the three light neutrinos turn out to be partially degenerate with masses around $0.02 - 0.1$ eV. This is close to the value of the mass parameter associated with neutrinoless double beta decay [41]. An important test of the model is the presence of doubly charged particles that should be found at the LHC. Another important feature is the prediction $\sin \theta_{13} \lesssim 0.01$. It would be of some interest to extend the discussion to larger gauge groups such as $SU(3)^3$ [29, 30, 45] and $SU(4)_c \times SU(2)_L \times SU(2)_R$ [11].

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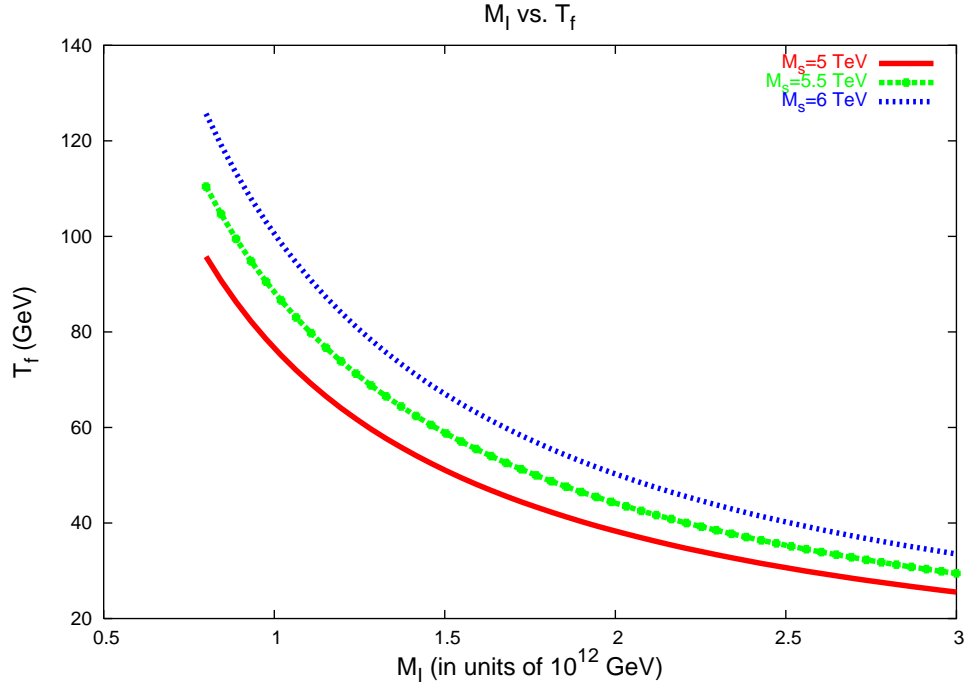
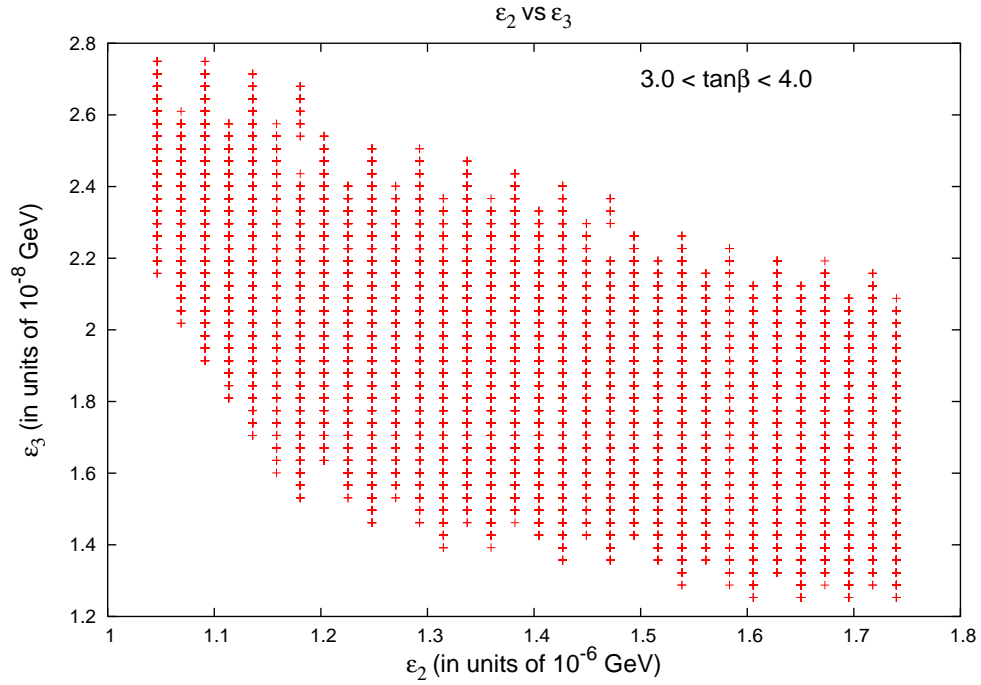FIG. 1: Variation of T_f with M_I for different values of M_s.

FIG. 2: The allowed region for ε_2 and ε_3 for $M_s = 5.5$ TeV and $M_I = 10^{12}$ GeV.
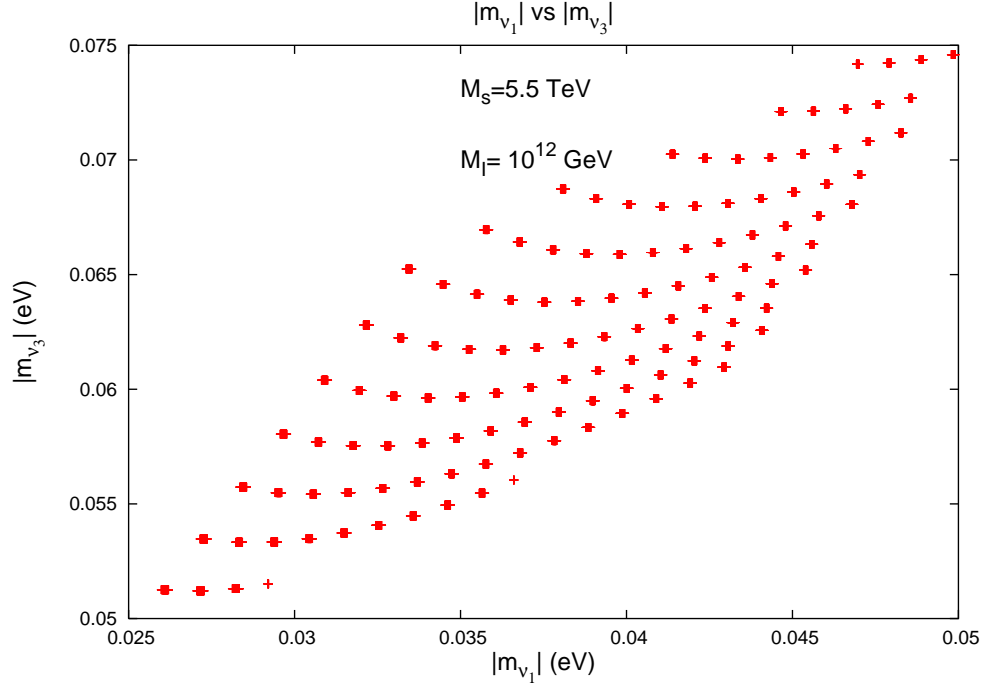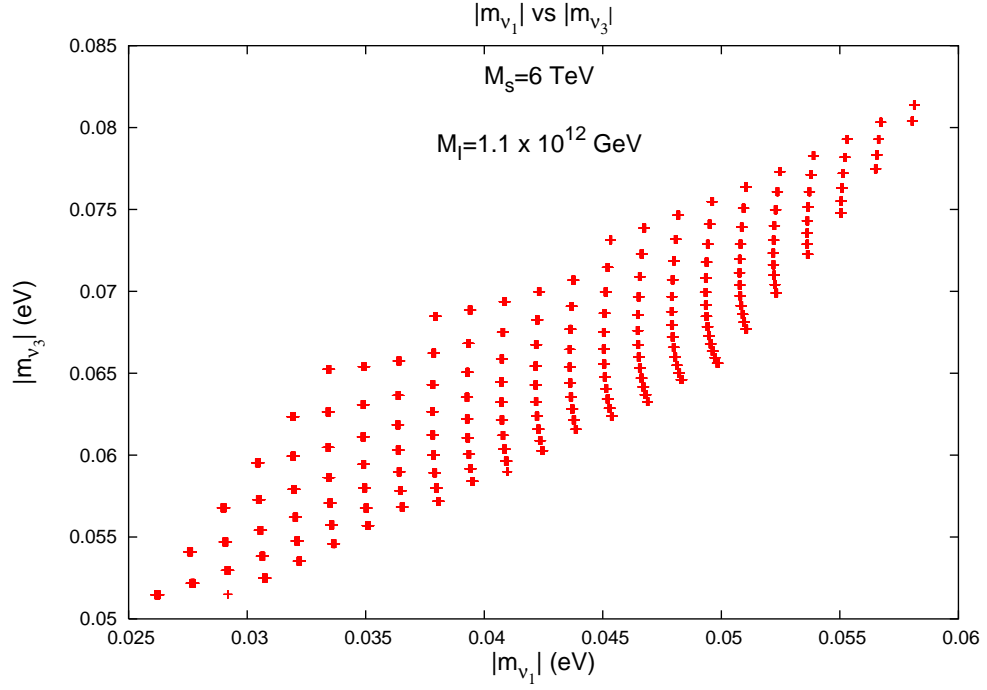
FIG. 3: The allowed region for $|m_{\nu_1}|$ and $|m_{\nu_3}|$.

FIG. 4: Same as Fig. (3), but for different M_s and M_I.